

Comments on "Controlling Discrete and Continuous Symmetries in Superradiant Phase Transitions with Circuit QED Systems"

Recently, the authors in Ref.[1] presented the $N = \infty$ solution of the $U(1)/Z_2$ Dicke model [2]. Here we point out that (1) The authors missed an important transformation relating the two parameter regimes, so their separate discussions on the two regimes is redundant. **Most importantly:** (2) Both $N = \infty$ classical limit and $1/N$ quantum fluctuations have been achieved in [2, 3]. It is the $1/N$ quantum fluctuations which lead to non-trivial new quantum phenomena. In view of only a few $N = 2 \sim 9$ qubits inside a circuit QED microwave cavity, they can be tested in near future experiments. (3) Several possible experimental implementations of the $U(1)/Z_2$ Dicke model have been proposed before and recently experimentally realized.

1. The authors in [1] claimed there are different physics in two different regimes $\Omega_E > \Omega_M$ and $\Omega_E < \Omega_M$. In fact, there is a transformation (see below) establishing the relation $\Omega_E \leftrightarrow \Omega_M$. So their separate discussions on the two regimes are redundant.

In the Eqn.1 called $U(1)/Z_2$ Dicke model in our work [2], g stands for the Rotating Wave (RW) term, and g' stands for the Counter-Rotating wave (CRW) term. It is straightforward to see that under $a \rightarrow e^{i\pi/2}a, \sigma_- \rightarrow e^{i\pi/2}\sigma_-$, the CRW term $g' \rightarrow -g'$. So in [2], we only focused on $0 < g' < g$ case. Obviously, $g' = 0$ is the $U(1)$ symmetry point, any $g' \neq 0$ breaks the $U(1)$ down to Z_2 . The authors in Ref.[1] re-wrote the Eqn.1 in [2] as their Eqn.1 with the straightforward relations $g = \Omega_E + \Omega_M, g' = \Omega_E - \Omega_M$. Under the same transformation, $\Omega_E \leftrightarrow \Omega_M$, so one only need to focus on $\Omega_E > \Omega_M$ at both $N = \infty$ and finite N . Below Eqn.13 in [2], we found the critical strength $g + g' = g_c = \sqrt{\omega_a \omega_b}$ which implies $\Omega_E = \sqrt{\omega_a \omega_b}/2$ shown in Fig.1 and 2 in [1].

2. The results shown in Fig.1-5 in [1] are just the $N = \infty$ (classical) limit of the effective action Eqn.12 at the order $1/N$ in [2]. It holds for any g and g' . Because the classical limit is technically trivial and not useful in any practical circuit QED system, it was only briefly mentioned below Eqn.13 in [2]. For example, the Fig.5c in [1] corresponds to $g' = 0$ in [2], namely, the $U(1)$ Dicke model. The Goldstone mode at $N = \infty$ is just the flat zero mode [2, 3], the amplitude model is nothing but the Higgs mode. Both modes are explicitly stressed in the title of [2].

The most important value of Eqn.12 is that it can be used to calculate quantum fluctuations at $1/N$ at any g and g' . In [2, 3], we computed the quantum fluctuations at order $1/N$ at the $U(1)$ limit $g' = 0$ and near the $U(1)$ limit $g'/g = \beta$ not too far away from the QCP. It is these quantum fluctuations which lead to highly interesting quantum phenomena. For example, they lift the flat zero (Goldstone) mode at $N = \infty$ to the oscillating shape shown in Fig.3a with the corresponding spec-

tral weights shown in Fig. 3b in [2]. The crucial Berry phase effects only show up at a finite N . It is the Berry phase which leads to the oscillating shape of the Goldstone mode shown in Fig.3a in [2]. All these important quantum effects get quenched in the $N = \infty$ (classical) limit. For example, the Goldstone mode is quenched to the flat zero mode [2, 3]. The amplitude mode shown in Fig.5b in [1] is nothing but the Higgs mode stressed in the title of and also fully discussed in [2]. We also computed the $1/N$ quantum fluctuations to the Higgs mode shown in Fig.5a and its spectral weight in Fig.5b [2]. Our $1/N$ quantum fluctuation results in Fig.3-4 match nearly perfectly well with the Exact diagonalization (ED) results for N gets as small as $N = 2$.

In a recent preprint [4], using Eqn.12, we investigated the quantum fluctuations at order $1/N$ at any $0 \leq g'/g = \beta \leq 1$ and full interaction strength.

3. The last part (about 1/10 of the paper) of Ref.[1] sketched a circuit QED implementation (Fig.6) of the $U(1)/Z_2$ Dicke model studied in [2]. In fact, the experimental implementations using both cold atoms inside a cavity and superconducting qubits inside a microwave cavity have been briefly discussed in [2, 3]. A recent experiment [5] realized the open version of the $U(1)/Z_2$ Dicke model by using cavity-assisted Raman transitions with cold atoms inside a cavity. We expect that the $U(1)/Z_2$ Dicke model can also be realized in superconducting qubits inside a microwave cavity. Then all the results on Goldstone and Higgs modes shown in Fig. 1-5 at and near the $U(1)$ limit in [2] and many other novel phenomena demonstrated in [4] at generic β can be detected for a few $N = 2 \sim 9$ qubits.

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- [1] Alexandre Baksic and Cristiano Ciuti, Phys. Rev. Lett. 112, 173601 (2014).
- [2] Yu Yi-Xiang, Jinwu Ye and W.M. Liu, Scientific Reports 3, 3476 (2013). It was cited in [1] as Ref.18.
- [3] Jinwu Ye and CunLin Zhang, Phys. Rev. A 84, 023840 (2011).
- [4] Yu Yi-Xiang, Jinwu Ye, W.M. Liu and CunLin Zhang, arXiv:1506.07030, version 2.
- [5] M. P. Baden, *et.al*, Phys. Rev. Lett. 113, 020408 (2014).